CHE 3804 NUCLEAR ENGINEERING

SECTION 2

NUCLEAR INTERACTIONS

NEUTRON INTERACTIONS

It has been shown in the previous sections that neutrons can be created by the integration of an electron with a proton. Furthermore a free neutron will in time disintegrate into a proton and an electron. Neutrons interact with the nuclei of atoms in various ways and may also be produced by the nuclei of certain atoms. The most common source of neutrons is the fissioning process where a heavy nucleus splits into two lighter nuclei. This fissioning of nuclei and the subsequent interaction of the resultant neutrons with other nuclei are the fundamental processes governing the production of power from nuclear energy. A knowledge of these processes is all important in the study of nuclear engineering.

FISSION

A heavy nucleus such as Uranium-235 will occasionally fission spontaneously into two lighter nuclei. A heavy nucleus such as this has about one and a half as many neutrons as protons in the nucleus. A mid-range nucleus however has only about one and a third as many neutrons as protons in its nucleus. Thus when a heavy nucleus fissions into two lighter nuclei not as many neutrons are required to maintain a stable configuration in the nucleus and some neutrons are rejected immediately the fission occurs. Generally two to three neutrons are emitted during the fission process.

In a nuclear reactor fissile nuclei such as Uranium-235 and Plutonium-239 are induced to fission by having their nuclei excited beyond the level of stability. This is done by subjecting them to the influence of a free neutron. Free neutrons interact with various nuclei in different ways causing a range of different reactions of which fission is just one. Most interactions involve scattering (non-absorption) or capture (absorption) of the neutrons. These reactions are important in maintaining and controlling the fission reactions in nuclear reactors.

Elastic Scattering (Elastic Collision)

Elastic scattering occurs when a neutron strikes a nucleus and rebounds elastically. In such a collision kinetic energy is transmitted elastically in accordance with the basic laws of motion. If the nucleus is of the same mass as the neutron then a large amount of kinetic energy is transferred to the nucleus. If the nucleus is of a much greater mass than the neutron then most of the kinetic energy is retained by the nucleus as it rebounds. The amount of kinetic energy transferred also depends upon the angle of impact and hence the direction of motion of the neutron and nucleus after the impact.

Inelastic Scattering (Inelastic Collision)

Inelastic scattering occurs when a neutron strikes and enters a nucleus. The nucleus is excited into an unstable condition and a neutron is immediately emitted but with a lower energy than when it entered. The surplus energy is transferred to the nucleus as kinetic energy and excitation energy. The excited nucleus subsequently returns to the ground state by the emission of a γ-ray. Such collisions are inelastic since all the initial kinetic energy does not reappear as kinetic energy. Some is absorbed by the nucleus and subsequently emitted in a different form (γ-ray). The emitted neutron may or may not be the one that initially struck the nucleus. In simplistic terms the neutron can be considered simply to be bouncing off an energy absorbing nucleus.

Radiative Capture

Radiative capture can be considered to be similar to the initial process leading to inelastic scattering. A neutron strikes and enters a nucleus. The nucleus is excited but the level of excitation is insufficient to eject a neutron. Instead all the energy is transferred to the nucleus as kinetic energy and excitation energy. The excited nucleus subsequently returns to the ground state by the emission of a γ-ray. The incoming neutron remains in the nucleus and the nuclide increases its number of neutrons by one. This is a very common type of reaction. It leads to the creation of heavier isotopes of the original element. Many of these may be radioactive and decay over time in different ways.

Nuclear Transmutation (Charged Particle Reaction)

Nuclear transmutation is similar to radiative capture and inelastic scattering. A neutron strikes and enters a nucleus. The nucleus is excited into an unstable condition but a particle other than a neutron is emitted. The emitted particles are either protons or α-particles. This leaves the nucleus still in an excited state and it subsequently returns to this ground state by the emission of a γ-ray. In this process the total number of protons in the nucleus is reduced by one for proton emission and by two for α-partial emission. The original element is thus changed or transmuted into a different element.

Neutron Producing Reaction

Neutron producing reactions occur when an additional one or two neutrons are produced from a single neutron. As before a neutron strikes and enters a nucleus. The nucleus is excited into an unstable condition as with inelastic scattering but two or three neutrons instead of only one neutron are emitted. The still excited nucleus subsequently returns to its ground state by the emission of a γ-ray.

Fission

Although spontaneous fission occasionally occurs fission is generally induced by neutrons. A neutron strikes and enters a heavy nucleus. The nucleus is excited into an unstable condition as with most of the foregoing interactions. In this unstable condition the nucleus splits into two new mid-range nuclei usually of unequal mass. Since these new nuclei do not need as many neutrons for stability some neutrons are emitted immediately. The surplus binding energy drives the new nuclei (fission fragments) and neutrons away from one another with high velocity. The new nuclei subsequently lose their kinetic energy and return to their ground states by emission of γ-rays. They are invariably still unstable with too many neutrons and subsequently decay usually by β-particle and γ-ray emission.

NEUTRON FLUX

Neutrons created by fission pass freely through solid material since atoms consist mainly of empty space. They have no charge and so are not affected by the charged electron cloud surrounding the nucleus. Furthermore the nucleus is so small compared with the size of the atom that the chance of the neutron colliding with it is relatively small. In a uniform material the neutrons travel randomly in all directions and some measure of their number or influence is required. A convenient parameter is *neutron flux*.

Neutron flux ϕ is defined as the number of neutrons per unit volume n multiplied by their velocity v.

$$\phi = n\,v$$

Neutron flux so defined has units of number per unit area per unit time. This can be considered as the number of neutrons passing through a particular cross sectional area per second.

If the neutrons travel in a parallel beam the area through which the neutrons pass may be at right angles to the beam and the given area will then be equal to the cross sectional area of the beam. This is the case in irradiation experiments where a beam of neutrons is directed out of the reactor through special ports.

Within the reactor the neutrons travel in all directions and the neutrons will pass through a given area in all directions and from both sides. This area is more difficult to define hence the definition of neutron flux as number multiplied by velocity.

MICROSCOPIC CROSS-SECTIONS

A solid material may be considered as being made up of tiny nuclei suspended in empty space. Each nucleus has an imaginary projected area which may interfere with the passage of a neutron. A neutron entering the solid will see these projected areas scattered everywhere but they are so small and so far apart that the chances of hitting one is practically nil. Eventually a neutron may hit a nucleus and will then interact with it in any of a number of possible ways. Other neutrons will simply pass it without any interaction.

It is interesting to note that the imaginary projected area or target area of a nucleus may be larger or smaller than the physical projected area as determined from the size of the actual nucleus. It may be larger because the nucleus has a sphere of influence surrounding it and any neutron passing within this sphere of influence may be attracted to interact with it. It may be smaller because some nuclei may allow neutrons to pass right through themselves without any interaction taking place. The imaginary projected area may thus be considered as being related to the probability of a reaction occurring. The larger the area the greater the probability of interaction.

It is also interesting to note that for different reactions with the nucleus there are different degrees of probability of interaction and therefore effectively different imaginary projected areas. Uranium-238 for example has a larger imaginary target area for elastic scattering than for radiative capture illustrating the greater probability of elastic scattering occurring. It is convenient for illustrative purposes to draw a pie diagram with the total area signifying the probability of all interactions occurring and each slice representing the probability of individual interactions taking place.

These imaginary projected areas are known as *nuclear cross-sections* and indicate the probability of any interaction occurring. The cross-sections of the nuclei of individual atoms are measured in square centimetres, square metres or barns where

$$1 \text{ barn} = 1 \times 10^{-24} \text{ cm}^2$$

$$1 \text{ barn} = 1 \times 10^{-28} \text{ m}^2$$

If the actual projected area of a nucleus is calculated it is found that for mid-range elements with an atomic mass number of about 90 this area is equal to 1 barn. Lighter elements have smaller projected areas and heavier elements larger projected areas.

A cross-section of 1 barn indicates immediately that the imaginary target area is roughly equal to the actual projected area of the nucleus. This allows cross-sections to be visualised. A cross-section of several hundred barn indicates that the nucleus has a large sphere of influence while a cross-section several magnitudes smaller than a barn indicates that the nucleus allows neutrons to pass through it with practically no chance of an interaction occurring.

There are different types of cross-sections, in fact there is one type of cross-section for each type of neutron interaction with the nucleus except for the relatively rare nuclear producing and nuclear transmutation reactions. The nomenclature for different cross-sections is given below with the different types of interactions

σ_s = Elastic scattering cross-section
σ_i = Inelastic scattering cross-section
$\sigma_{n,\gamma}$ = Radiative capture cross-section
σ_a = Absorption cross-section
σ_f = Fission cross-section

Values for these are tabulated but are often combined into two main types of interactions:

σ_s = Scattering cross-section
σ_a = Absorption cross-section

When these are combined they are added together so that the scattering cross-section includes both elastic and inelastic scattering and the absorption cross-section includes both radiative capture and fission.

MACROSCOPIC CROSS-SECTIONS

The *macroscopic cross-section* Σ is the cross-section density in a material. It is defined as the number of nuclei per unit volume N multiplied by the *microscopic cross-section σ.* The units are the inverse of length (cm^{-1} or m^{-1})

$$\Sigma = N \sigma$$

This provides a basis for the comparison of different materials. A dense material with nuclei of small cross-section would be seen by neutrons to be effectively the same as a rare material with nuclei of large cross-section.

REACTION RATE

Since the macroscopic cross-section Σ is effectively the material parameter seen by the neutrons and since neutron flux ϕ is effectively the number of neutrons passing through a given place per unit time it follows then that the reaction rate R between neutrons and nuclei is given by:

$$R = \Sigma \phi$$

This may also be written as

$$R = N \sigma n v$$

This is perfectly logical since the reaction rate R would likely be proportional to the number of nuclei N, the cross-section σ, the number of neutrons n and the velocity of the neutrons v. At a high velocity a neutron would have more chances of meeting a nucleus than at a low velocity.

SUMMARY

The following relationships with units are summarised below:

Macroscopic cross-section

N = nuclei per unit volume	(nuclei / cm^3)
σ = microscopic cross-section	(cm^2)
$\Sigma = N \sigma$	(cm^{-1})

Neutron Flux

n = neutrons per unit volume (neutrons / cm^3)
v = neutron velocity
(cm / s)

$\phi = n\, v$ (neutrons / cm^2 s)

Reaction Rate

ϕ = neutron flux (neutrons / cm^2 s)
Σ = macroscopic cross-section (cm^{-1})

$R = \phi\, \Sigma$ (reactions / cm^3 s)

NEUTRON ATTENUATION

When a beam of neutrons impinges upon a solid body the neutrons interact with nuclei within the body. Those not interacting continue through the body. As the beam progresses through the body more and more interactions occur and less and less neutrons continue on through the material. The beam of neutrons diminishes in intensity and is attenuated by the material.

The decrease in intensity dI over any section of material is proportional to the neutron beam intensity I, microscopic cross-section of the material σ, number density of nuclei N and the thickness of the material dx

$$dI = -\, I\, \sigma\, N\, dx$$

If the macroscopic cross section Σ is used this becomes:

$$dI = -\, I\, \Sigma\, dx$$

The solution to this differential equation is

$$I = I_o\, e^{-\Sigma x}$$

This is the equation for the attenuation of a neutron beam. The attenuation of a γ-ray beam is similarly:

$$I = I_o e^{-\mu x}$$

Here μ is the *attenuation coefficient* of the γ-ray beam

MEAN FREE PATH

The *neutron mean free path* λ is defined as the average distance a neutron travels without interacting with a nucleus. The reaction rate R is equal to the macroscopic cross section Σ multiplied by the neutron flux ϕ

$$R = \Sigma \phi$$

$$R = \Sigma n v$$

The reaction rate R can also be written in terms of the number of neutrons n multiplied by their velocity v and divided by their mean free path λ.

$$R = n v / \lambda$$

This in effect states that more reactions will occur when the velocity is higher and the mean free path lower. If these two equations for reaction rate are combined then the following is obtained:

$$\Sigma n v = n v / \lambda$$

$$\lambda = 1 / \Sigma$$

Thus the mean free path λ is the inverse of the macroscopic cross-section Σ.

This can also be proven by considering the total (integrated) distance travelled by all neutrons before interacting and dividing by the number of neutrons initially in the beam.

SCATTERING CHARACTERISTICS

It was seen previously that, with elastic scattering, the neutron rebounded from a nucleus with kinetic energy conserved and no excitation of the nucleus. Furthermore, with inelastic scattering, the neutron interacted with the nucleus leaving it in an excited state.

Both of these scattering effects may occur in a single nuclide and it is found that the probability of these reactions is, to a large degree, dependent upon the energy of the incoming neutron.

When at very low energies, the neutron does not interact with the nucleus and is scattered as if influenced by the physical size of the nucleus. The apparent area A of the nucleus for such scattering is given by:

$$A = 4\,\pi\,R^2$$

The radius of the nucleus R is in turn given in terms of atomic mass number A by the following:

$$R = 1.25 \times 10^{-13}\,A^{1/3} \qquad \text{(cm)}$$

The apparent area of the nucleus for such scattering is the neutron scattering cross-section σ_s

$$\sigma_s = 4\,\pi\,R^2$$

This scattering at low neutron energy is called *potential scattering* and is constant over a range of low neutron energies.

When at intermediate energies some neutrons have an energy that raises the nucleus to a discrete excitation level. Under these conditions absorption and subsequent emission of a neutron occurs more easily. Since the nucleus is left in an excited state the emitted neutron is at a lower energy. This results in inelastic scattering. If there is no match in vibrational characteristics absorption does not occur easily. This results in widely varying scattering probabilities over a certain range of neutron energies. This is called the *resonance region.*

When at very high energies there is no longer a match in vibrational characteristics and the probability of scattering falls with increasing energy as those neutrons passing close to the nucleus are less affected by it. This is known as the *smooth region.*

ABSORPTION CHARACTERISTICS

It was seen previously that, with both inelastic scattering and radiative capture, the neutron interacted with the nucleus leaving it in an excited state. Both of these interactions may occur in a single nuclide and it is found that the probability of these reactions is to a large degree dependent upon the energy of the incoming neutron.

For many nuclides there is a threshold neutron energy above which inelastic scattering occurs and below which radiative capture occurs. This is due to the fact that the neutron brings with it a certain amount of energy which is transferred to the nucleus when it enters the nucleus. If the neutron energy is sufficient to raise the energy of the nucleus above the threshold value then the excited nucleus can emit a neutron along with a γ-ray. If the energy of the excited nucleus remains below the threshold value no neutron will appear and only a γ-ray will be emitted. The threshold energy corresponds with the binding energy of the additional neutron while the γ-ray corresponds with the amount of energy remaining above the ground state of the nucleus. High velocity (high energy) neutrons are thus likely to be elastically scattered while low velocity (low energy) neutrons likely to suffer radiative capture.

RADIATIVE CAPTURE MODELS

From the above it is evident that radiative capture is likely to occur with neutrons below the threshold energy value that is with lower velocity neutrons. As the velocity is decreased further it is found, for many nuclides, that the probability of radiative capture increases. This probability is in fact inversely proportional to the velocity (square root of energy).

This can be visualised by imagining that the nucleus has a sphere of influence around it. A neutron passing through this sphere of influence will spend a certain period of time within that sphere of influence. For a given path the higher its velocity the shorter the time spent within the sphere of influence. If the probability of capture is proportional to the time spent within the sphere of influence then the probability of capture (absorption cross-section σ_a) will be inversely proportional to velocity v

$$\sigma_a \propto 1 / v$$

CROSS-SECTIONS

The above may be summarised and illustrated by plotting on a composite diagram. The elastic scattering cross-section σ_s is constant in the low energy potential region, fluctuates in the resonance region and falls slowly with increasing energy in the smooth region. The inelastic cross-section σ_i is only apparent above a certain threshold energy. The radiative capture cross-section σ_γ is inversely proportional to velocity in the *1/v region*, fluctuates in the resonance region and drops to a low value or disappears at high energies. The total cross-section σ_t is a summation of all the individual cross-sections. Note that both the cross-section and neutron energy are plotted on logarithmic scales.

LOG MEAN ENERGY DECREMENT

When neutrons interact with nuclei in elastic scattering collisions they lose energy. The amount of energy lost depends upon the mass of the nucleus and the angle of incidence of the neutron.

The minimum energy E_{min} after one collision is:

$$E_{min} = \alpha\, E_o$$

$$\alpha = [(A - 1) / (A + 1)]^2$$

Considering the results of various angles of incidence it is found that the average energy E_{ave} after one collision is:

$$E_{ave} = (½)\,(1 + \alpha)\, E_o$$

Average energy loss after one collision is given by:

$$\Delta E = E_o - E_{ave}$$

$$\Delta E = (½)\,(1 - \alpha)\, E_o$$

$$\alpha = [(A - 1) / (A + 1)]^2$$

The *logarithmic mean energy decrement* ξ is the average of the difference of the logarithmic energy values:

$$\xi = [\mathrm{Ln}\, E_o - \mathrm{Ln}\, E]_{average}$$

$$\xi = [- \mathrm{Ln}\, (E / E_o)]_{average}$$

The value of the logarithmic mean energy decrement for any isotope of atomic mass number A:

$$\xi = 1 + [(A - 1)^2 / 2\, A]\, \mathrm{Ln}\, [(A - 1) / (A + 1)]$$

An approximate value for the logarithmic mean energy decrement is given by the following empirical equation:

$$\xi = 2 / [A + (2/3)]$$

The number of elastic collisions N required for the neutron energy to drop from an initial energy E_i to a final energy E_f is given by:

$$N \xi = \text{Ln}(E_i / E_f)$$

The value of N for a high energy neutron from fission to become thermalised at ambient conditions is 18 for Hydrogen, 43 for Helium and 115 for Carbon. Ligher elements are efficient at reducing neutron energy because they are light and absorb a lot of energy when struck by a nucleus.

DEFINITIONS

Mean Logarithmic Energy Decrement ξ

$N \xi = \text{Ln}(E_i / E_f)$

N = number of collisions

E_i = initial energy (2 MeV after fission)

E_f = final energy (0.025 eV when thermalised)

Macroscopic Scattering Cross-section Σ_s

$\Sigma_s = N \sigma_s$	(cm^{-1})
N = nuclei per unit volume	(nuclei / cm^3)
σ_s = microscopic scattering cross section	(cm^2)

Slowing Down Power

$$\text{slowing down power} = \xi\, \Sigma_s \qquad (\text{cm}^{-1})$$

Moderating Ratio

$$\text{moderating ratio} = \xi\, \Sigma_s / \Sigma_a$$

FISSION AND FUSION

It has been shown that both the fission of light elements and the fission of heavy elements will produce energy. This is due to the fact that the curve of binding energy per nucleon is less for light and heavy elements than for mid-range elements. The amount of energy released can be calculated from the mass defect if the final products are known. For fusion a range of different reactions is possible as Hydrogen fuses into Helium. For fission only one reaction is possible for any particular fuel but a range of fission products is produced. On average about 200 MeV is produced from a fission reaction.

FISSION

During the fission process a number of neutrons is released since otherwise the resulting fission products would have too many neutrons. Even so they have an excess of neutrons and decay towards a more stable condition. These neutrons are free to enter other fissile nuclei and so cause further fissions to maintain a chain reaction. If the same number of neutrons continues into the next generation the chain reaction is stable. To achieve this some neutrons must be captured without producing fission since for every neutron causing fission on average two or three are produced.

Fission occurs spontaneously in some heavy nuclides but is rare. This contributes to the gradual decay of the nuclide and creates a few free neutrons within the fuel. This is an important factor when loading new fuel into a reactor as the resulting low level nuclear chain reactions could inadvertently grow out of control. Fission induced by neutrons is due to the fact that the incoming neutron adds sufficient energy to the nucleus to raise its energy level enough for it to become unstable. Nuclides that fission when unstable are known as *fissile* materials. There are four such fissile isotopes:

Uranium-233
Uranium-235

Plutonium-239
Plutonium-241

A number of other nuclides will fission if the incident neutron has a high kinetic energy. This kinetic energy together with the binding energy can raise the energy level of the nucleus sufficiently for it to become unstable and to fission. Such nuclides are known as *fissionable* materials. Fissionable isotopes thus require energetic neutrons to cause fission and as such are nonfissile.

FISSION CHARACTERISTICS

Uranium-235 and Uranium-238 have scattering and absorption cross-sections similar to other materials. In U-235 absorption usually leads to fission and in the low neutron energy region the absorption cross-section is very high but decreases with increasing neutron energy since it is inversely proportional to the neutron energy. There is then a resonance region where there are peaks with a high probability of absorption. At high energies there is a low probability of absorption and hence fission and the cross-section is low. In U-238 absorption does not lead to fission except at very high neutron energies. At low neutron energies there is a low probability of absorption and this is also inversely proportional to neutron energy. In the resonance region however there are very high peaks of absorption. The absorption cross-section then falls again to low values in the high energy region. At very high energies absorption leads to fission.

FISSION PRODUCTS

During fission two fission fragments usually of unequal mass are produced. These generally have atomic mass numbers of between 100 and 140 though a range of possibilities exists from an atomic mass number of about 70 to about 160. The amount of a particular fission product occurring is known as the *fission yield.* Fission yields vary for different fissile materials and for fission with higher energy neutrons. The fission yields of Plutonium-239, for example, show that somewhat more fission products of intermediate mass number are produced than is the case with Uranium-235. For high energy neutrons the fission yield curve is much flatter still with even more fission products of intermediate mass being produced.

NEUTRON ENERGY SPECTRUM

Neutrons produced at the time of fission are known as *prompt neutrons* . Some neutrons appear a short time later and these are known as *delayed neutrons* . The prompt neutrons are produced with a range of different energies. Most energy from fission appears as kinetic energy of the heavy fission products but some is carried away by the neutrons also as kinetic energy. The energy of prompt neutrons varies from about zero to about 8 MeV. If a sample of 100 prompt neutrons is analysed it is found that some 35 have an energy of about 1 MeV the most probable energy while the average energy is about 2 MeV. The results are usually plotted as a smooth curve of fraction emitted versus neutron energy.

DELAYED NEUTRONS

Delayed neutrons are emitted from some fission products a short while after fission has occurred. Most fission products are unstable and decay towards a more stable state by emitting particles, usually β-particles to convert a neutron into a proton. Some however are sufficiently unstable to emit neutrons directly or subsequently (after β-particle emission) to reduce the neutron number. An example is the fission product Bromine-87. This decays to Krypton-87 by the emission of a β-particle and then to Krypton-86 by the emission of a neutron. The half-lives for these reactions are so short that the neutrons appear almost immediately but the time lag is sufficiently important to have a very marked influence on the control of nuclear reactors. The delay is long enough to be detected by control systems which can respond in time to changes in delayed neutron production. No control system can respond in time to changes in prompt neutron production.

Delayed neutrons come from some twenty fission products or *delayed neutron precursors*. Each precursor produces a neutron following decay or decays of different half-lives. For convenience these are grouped into six groups of precursors such that each group produces neutrons following decay according to a particular half-life. The first group has a half-life of 55 seconds while the last group has a half-life of only 0.2 second. Each group has a different yield of neutrons per fission with the fourth group producing nearly 40% while the first and last groups produce only about 3% and 4% respectively. Overall the total yield of delayed neutrons is only 0.65% of all neutrons produced in fission. This small amount is very important in the control of nuclear reactors as the control system must be able to detect small enough changes to maintain control on delayed neutrons.

FISSION PROCESS SUMMARY

For fission to occur the incoming neutron must add sufficient energy to the fissile nucleus to raise its energy above the critical value for fissioning. Low energy neutrons however interact more readily with Uranium-235 to cause fission than do high energy neutrons.

Uranium-238 on the other hand will only undergo fission with high energy neutrons. The shape of the neutron-proton ratio curve results in additional neutrons being produced in fission. Neutrons produced in fission have a range of energies with an average of about 2 MeV. The energy produced in one fission process is about 200 MeV. These high energy neutrons must be slowed down or *moderated* to reduce their energy so as to be able to interact easily with further Uranium-235 nuclei to start a new cycle.

CHARGED PARTICLES

Fission products are produced as a light fragment and a heavy fragment from each fission. The lighter fragments have kinetic energies of about 100 MeV while the heavier fragments have energies of about 70 MeV. This division of energies arises from the conservation of momentum as two initially stationary parts of different mass recoil from one another. These fission fragments leave behind some twenty electrons and immediately become positively charged. They lose kinetic energy rapidly in the surrounding material producing heat and ionisation along their path. Their range is very short being in the order of 1.4×10^{-3} cm (0.014 mm) in Uranium dioxide fuel (U_3O_8)

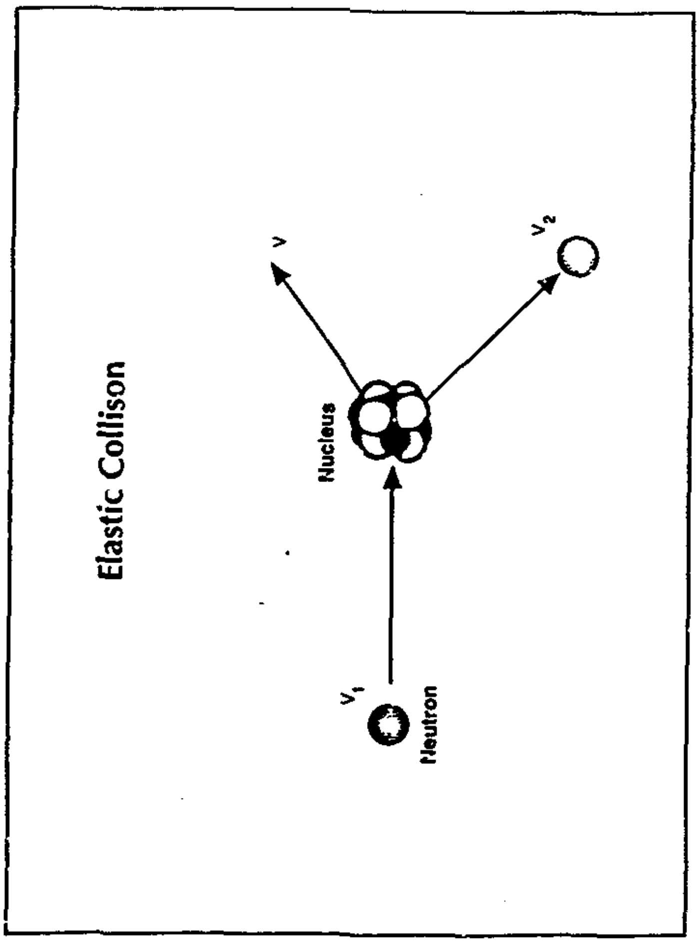
Alpha particles also interact with other atoms causing ionisation. They travel in a short straight path with a range dependent upon their energy according to the following formulae.

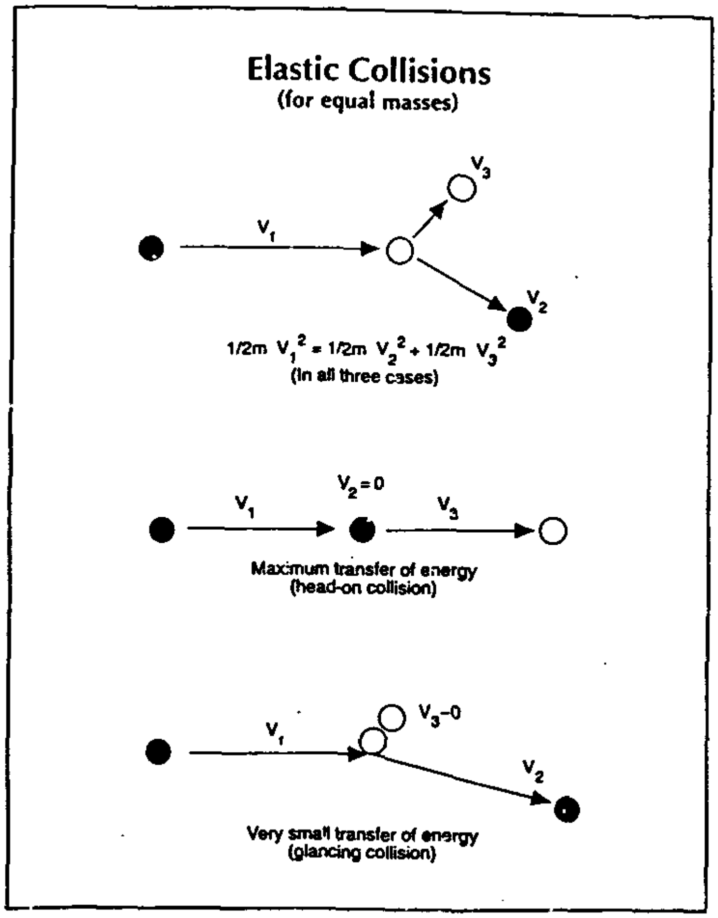
$$R_{air} = f\,(\text{Energy})$$

$$R_{medium} = R_{air}\,(\rho_{air} / \rho_{medium})\,(M_{medium} / M_{air})^{1/2}$$

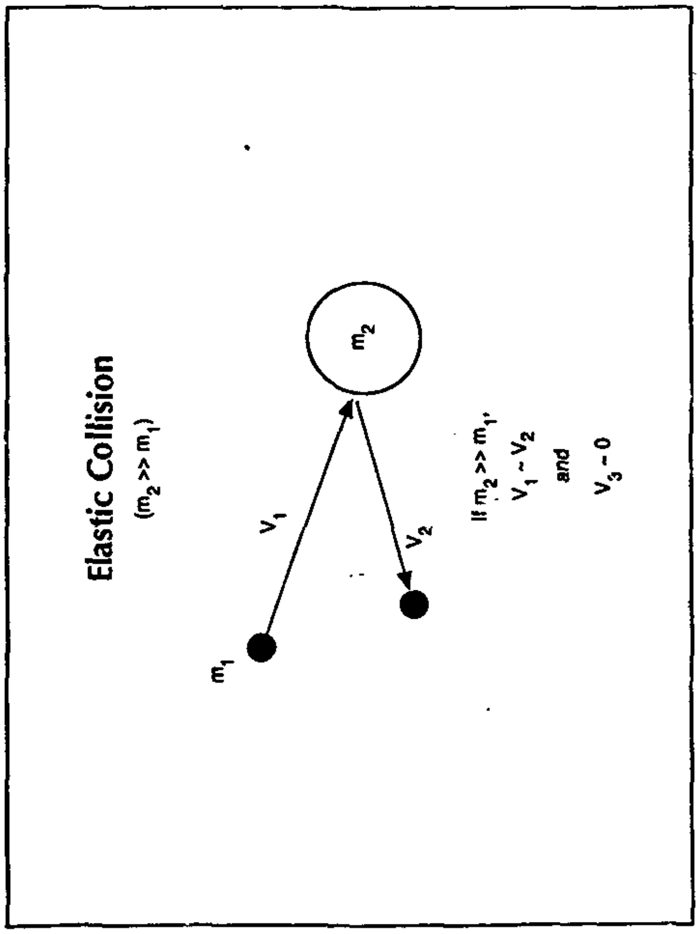
Beta particles travel in a zigzag path and are not very penetrating since they are very light. Their range is also a function of their energy

$$R_{max} = f\,(\text{Energy}) / \rho_{medium}$$

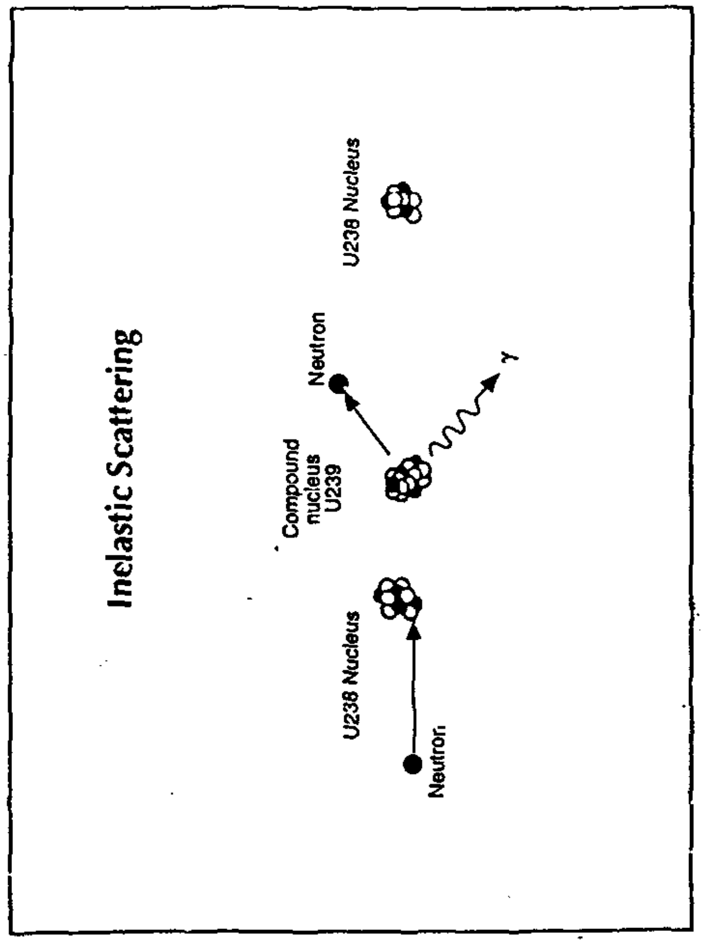

Elastic Collison
V
V2
Nucleus
V1
Neutron


Elastic Collisions
(for equal masses)
V1
V3
V2
$1/2m\ V_1^2 = 1/2m\ V_2^2 + 1/2m\ V_3^2$
(In all three cases)
V1
V2 = 0
V3
Maximum transfer of energy
(head-on collision)
V1
V3~0
V2
Very small transfer of energy
(glancing collision)

OH 2.1

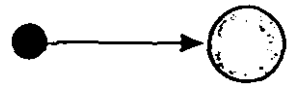

Elastic Collision
(m2 >> m1)
m1
V1
V2
m2
If m2 >> m1,
V1 ~ V2
and
V3 ~ 0

OH 2.2



Inelastic Scattering
(internal reaction)

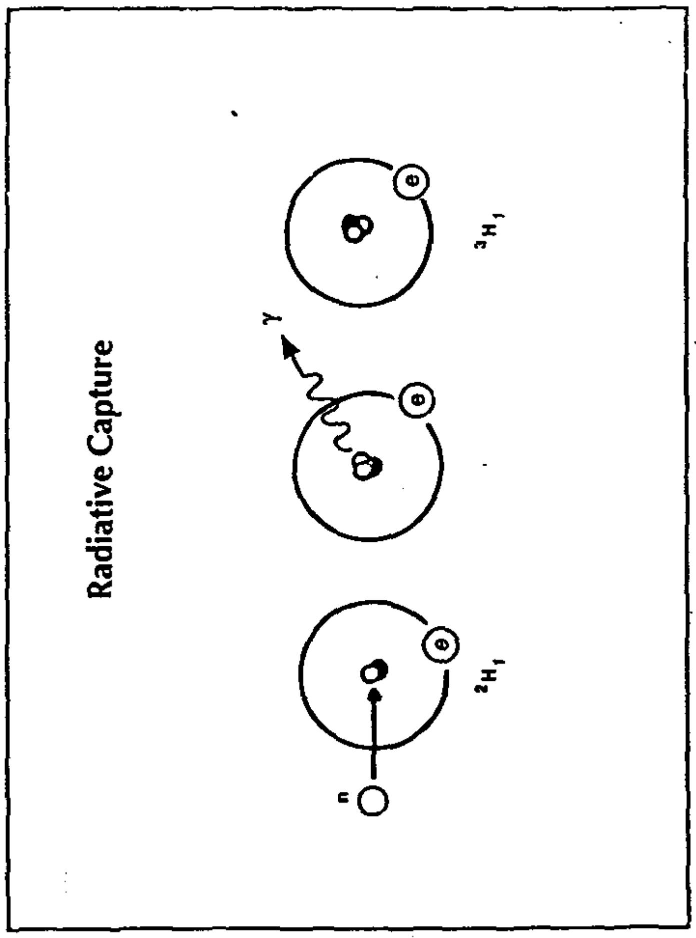
1. Neutron enters stable nucleus



2. Nucleus is excited

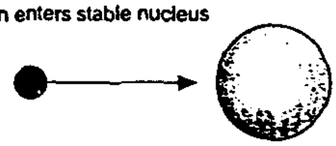
3. Neutron and γ - photon are emitted

4. Nucleus is stable again



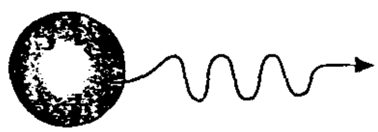
Radiative Capture

1. Neutron enters stable nucleus



$$^{1}n_{0} + {}^{238}U_{92} \longrightarrow {}^{239}U^{*}_{92}$$

2. Gamma ray leaves excited nucleus



$$^{239}U^{*}_{92} \longrightarrow {}^{239}U_{92} + \gamma$$

This may be written as

$$^{1}n_{0} + {}^{A}X_{Z} \longrightarrow {}^{A+1}X_{Z} + \gamma$$

$$^{238}U\ (n,\gamma)\ {}^{239}U$$

$^{239}U_{92}$ subsequently transmutes to $^{239}Pu_{94}$. How?

Transmutation

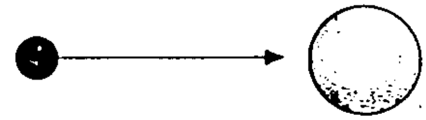
^{1}Neutron$_0$ + $^{16}O_8$ Nucleus Becomes $^{16}N_7$ Nucleus + ^{1}Proton$_1$

This reaction may be written as

$$^1n_0 + {}^{16}O_8 \longrightarrow {}^{16}N_7 + {}^1p_1$$

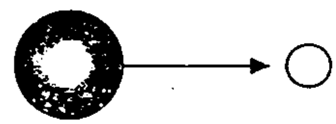
Nuclear Transmutation

1. Neutron enters stable nucleus



^{1}Neutron$_0$ + $^{16}O_8$ Nucleus $\longrightarrow$ $^{17}O_8$ Nucleus

2. Another particle leaves excited nucleus



$^{17}O_8$ Nucleus $\longrightarrow$ $^{16}N_7$ Nucleus + ^{1}Proton$_1$

This may be written as

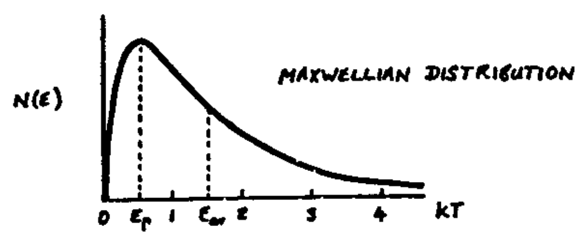
$$^1n_0 + {}^{16}O_8 \longrightarrow {}^{16}N_7 + {}^1p_1$$

$^{16}O\,(n,p)\,^{16}N$

(^{16}N nucleus subsequently emits gamma rays)

NEUTRON ENERGY

ENERGY DISTRIBUTION



BOLTZMANN'S CONSTANT

$k = 1.38 \times 10^{-23}$ J/°K

$k = 8.62 \times 10^{-5}$ eV/°K

MOST PROBABLE ENERGY

$E_p = \frac{1}{2}$ kT

AVERAGE ENERGY

$E_{av} = \frac{3}{2}$ kT

DEFINITION OF NEUTRON ENERGY ASSUMES THAT ALL NEUTRONS ARE AT SAME VELOCITY (ENERGY)

$E = kT$

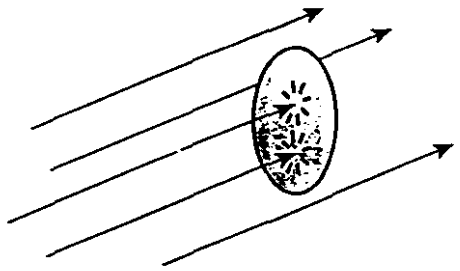
Neutron Flux

Neutron flux ϕ is defined as the number of neutrons n multiplied by their velocity v

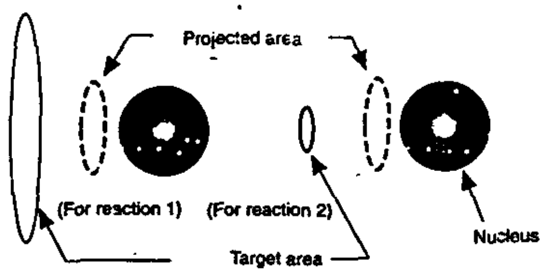
$$\phi = nv \qquad \frac{\text{number}}{\text{cm}^3} \times \frac{\text{cm}}{\text{s}} = \frac{\text{number}}{\text{cm}^2\,\text{s}}$$

Since neutron flux ϕ has units of $cm^{-2}s^{-1}$ it can be considered as the number of neutrons passing through a particular cross sectional area per unit time

Target Areas



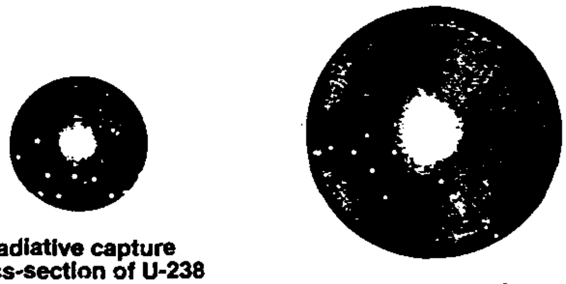
Target area may be smaller or larger than projected (physical) area of nucleus



Target areas are different for different nuclear reactions

OH 3.2

Cross-Sections for U-238



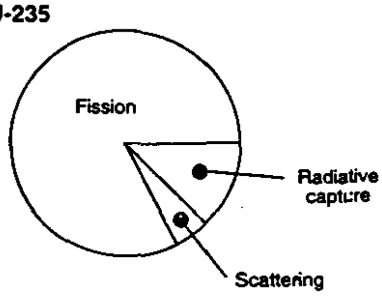
Radiative capture Cross-section of U-238

Elastic scattering Cross-section of U-238

Fig. 3.1

Pie Diagram for Cross-Sections of U-235 and Nat–U

U-235



Nat-U

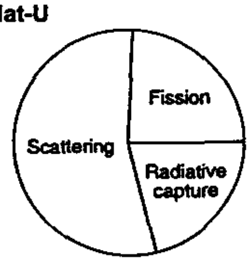


Fig. 3.2

Comparison of Cross-Sections for Thermal Neutrons

Element	Nucleus radius $r = 1.2 \times 10^{-15} A^{1/3}$ (m)	Nucleus radius (cm)	Nucleus area $A = \pi r^2$ (cm^2)	Cross section σ_a (cm^2)	Cross section σ_s (cm^2)
H	0.0012×10^{-12}	0.12×10^{-12}	0.045×10^{-24}	0.332×10^{-24}	38×10^{-24}
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.6×10^{-12}
C	0.0027×10^{-12}	0.27×10^{-12}	0.229×10^{-24}	0.0034×10^{-24}	4.75×10^{-24}
O	[illegible]	0.30×10^{-12}	[illegible]	0.00027×10^{-24}	3.76×10^{-24}
Pb	0.0071×10^{-12}	0.71×10^{-12}	1.584×10^{-24}	0.17×10^{-24}	11.4×10^{-24}
[illegible]	[illegible]	[illegible]	[illegible]	7.53×10^{-24}	8.9×10^{-24}
U-235	0.0074×10^{-12}	0.74×10^{-12}	1.720×10^{-24}	99×10^{-24} (n,γ) 582×10^{-24} (n,f)	

Note that projected area of nucleus is about 1×10^{-24} cm = 1 Barn

OH 3.4

Thermal Neutron Cross-Sections of Fuel Atoms (in Barns)

Taken from Atomic Energy Review (IAEA), 1969, Vol 7, No 4, p.3

	σ_f	$\sigma_{n,\gamma}$	σ_a	σ_s	ν	σ_f/σ_a (%)
U-233	530.6	47.0	577.6	10.7	2.487	92
U-235	[illegible]	[illegible]	678.5	[illegible]	2.430	86
U-238	0	2.71	2.71	~10	0	
Nat-U	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Pu-239	741.6	271.3	1012.9	8.5	2.890	73
Pu-241	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

Definitions

Macroscopic cross-section

(Cross-section density in material)

$\Sigma = N\sigma$ $\left(\frac{1}{cm}\right)$ or (cm^{-1})

N = Nuclei per unit volume $\left(\frac{nuclei}{cm^3}\right)$

σ = Microscopic cross-section (cm^2)

Neutron flux

(Neutrons passing through given area per second)

$\phi = nv$ $\left(\frac{neutrons}{cm^2\,s}\right)$

n = Neutrons per unit volume $\left(\frac{neutrons}{cm^3}\right)$

v = Neutron velocity $\left(\frac{cm}{s}\right)$

Reaction rate

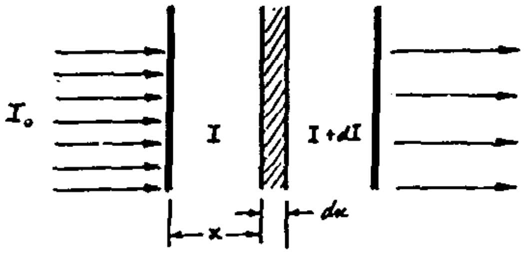
(Reaction rate of neutrons with material)

$R = \phi\Sigma$ $\left(\frac{reactions}{cm^3\,s}\right)$

ϕ = Neutron flux $\left(\frac{neutrons}{cm^2\,s}\right)$

Σ = Macroscopic cross-section $\left(\frac{1}{cm}\right)$

NEUTRON ATTENUATION



A = FRONTAL AREA (m^2)

σ = MICROSCOPIC CROSS SECTION $(m^2/\text{nucleus})$

N = ATOM DENSITY (nuclei/m^3)

$NA\,dx$ = NUMBER OF ATOMS IN SLAB (nuclei)

$\sigma NA\,dx$ = EFFECTIVE CROSS SECTIONAL AREA (m^2)

EFFECTIVE AREA / ACTUAL AREA = $\sigma NA\,dx / A$

∴ PROBABILITY OF INTERACTION = $\sigma NA\,dx/A = \sigma N dx$

∴ DECREASE IN INTENSITY OF BEAM

$$dI = -I\sigma N dx$$
$$\frac{dI}{I} = -\sigma N dx$$
$$\int_{I_0}^{I}\frac{dI}{I} = -\sigma N\int_0^x dx$$

$$\int_{I_0}^{I}\frac{dI}{I} = -\sigma N\int_0^x dx$$
$$\ln\left(\frac{I}{I_0}\right) = -\sigma N x$$
$$\frac{I}{I_0} = e^{-\sigma N x}$$
$$I = I_0\, e^{-\sigma N x}$$
$$I = I_0\, e^{-\Sigma x}$$

WHERE Σ IS THE MACROSCOPIC CROSS SECTION

DIFFERENTIATING

$$\frac{dI}{dx} = I_0(-\Sigma)e^{-\Sigma x}$$
$$dI = -I_0\,\Sigma\, e^{-\Sigma x}\,dx$$

NEUTRONS IN THIS BEAM HAVE TRAVELLED A DISTANCE x WITHOUT INTERACTING. FOR AN INFINITE SLAB THE TOTAL DISTANCE TRAVELLED BY ALL NEUTRONS IS :

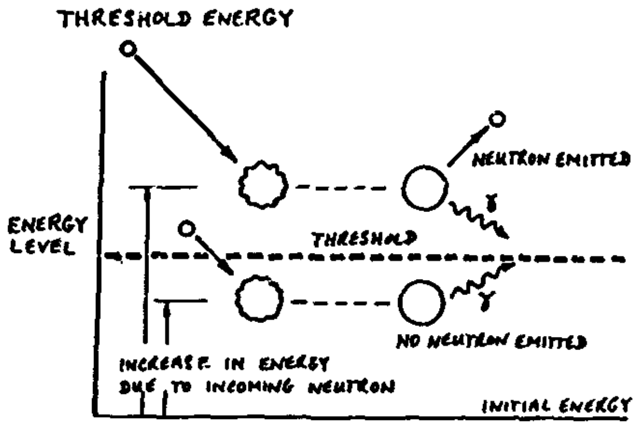
$$-\int_{x=0}^{x=\infty} x\,dI = I_0\Sigma\int_0^\infty x\,e^{-\Sigma x}dx$$

THE MEAN FREE PATH λ IS THIS TOTAL INTERACTION DIVIDED BY THE ORIGINAL BEAM INTENSITY.

MEAN FREE PATH :

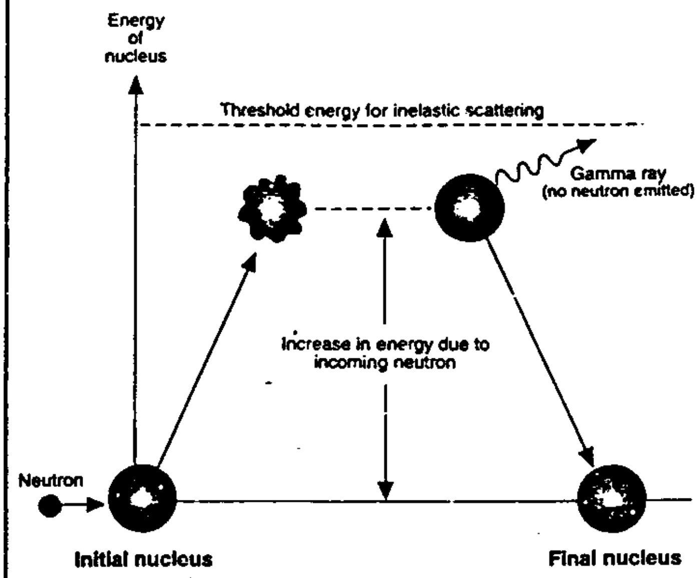
$$\begin{aligned}\lambda &= \frac{I_0\Sigma\int_0^\infty x e^{-\Sigma x}dx}{I_0}\\ &= \Sigma\int_0^\infty x e^{-\Sigma x}dx\\ &= \Sigma\left[x\left(\frac{1}{-\Sigma}\right)e^{-\Sigma x} - \frac{1}{(-\Sigma)^2}e^{-\Sigma x}\right]_0^\infty\\ &= \Sigma\left[e^{-\Sigma x}\left\{\frac{x}{-\Sigma} - \frac{1}{\Sigma^2}\right\}\right]_0^\infty\\ &= \Sigma\left[\frac{1}{\Sigma^2}\right]\\ &= \frac{1}{\Sigma}\end{aligned}$$

INELASTIC SCATTERING

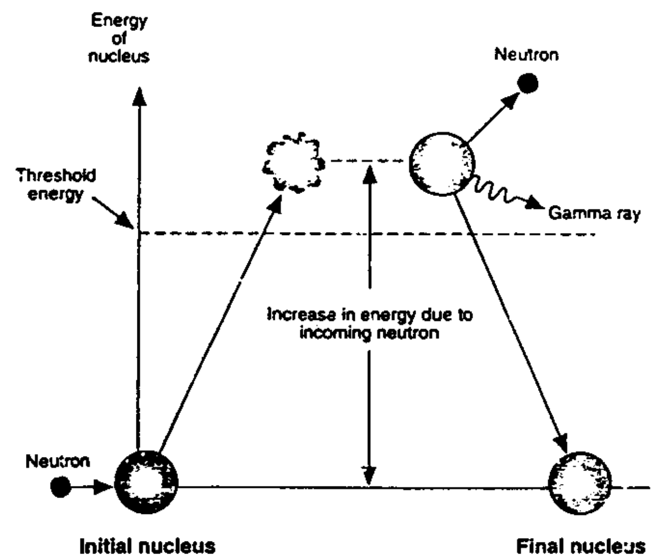


NEUTRONS MUST HAVE A CERTAIN THRESHOLD ENERGY TO CAUSE INELASTIC SCATTERING (EMISSION OF NEUTRON)

Radiative Capture
(below threshold energy)

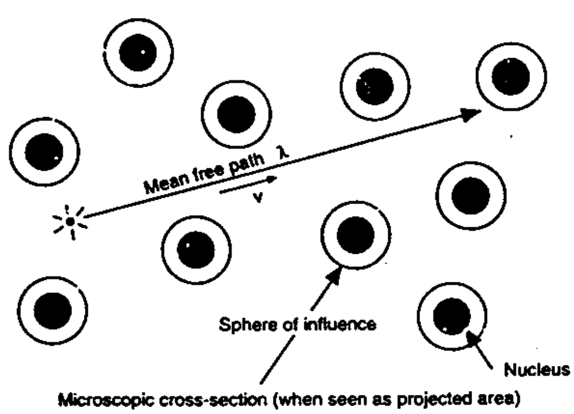


Inelastic Scattering
(above threshold energy)



Neutron Mean Free Path

Macroscopic cross-section gives density of microscopic cross-sections of atoms in material



$$R = \Sigma\phi = \Sigma nv \text{ -------- } (1)$$

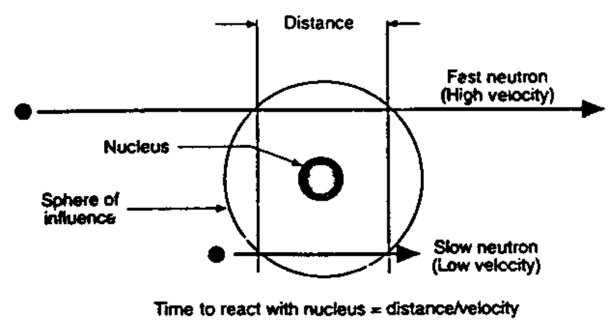
But R = Number of neutrons $\times \dfrac{\text{Distance}}{\text{Second}} \times \dfrac{1}{\text{Mean distance}}$

$$R = \frac{nv}{\lambda} \text{ -------------- } (2)$$

From (1) And (2)

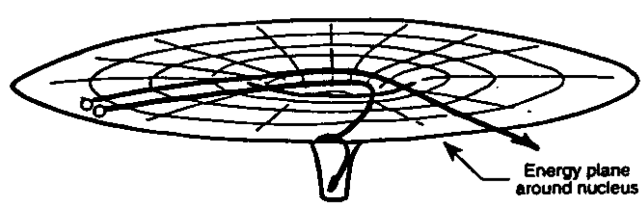
$$\lambda = \frac{1}{\Sigma} \text{ cm}$$

Interaction Models



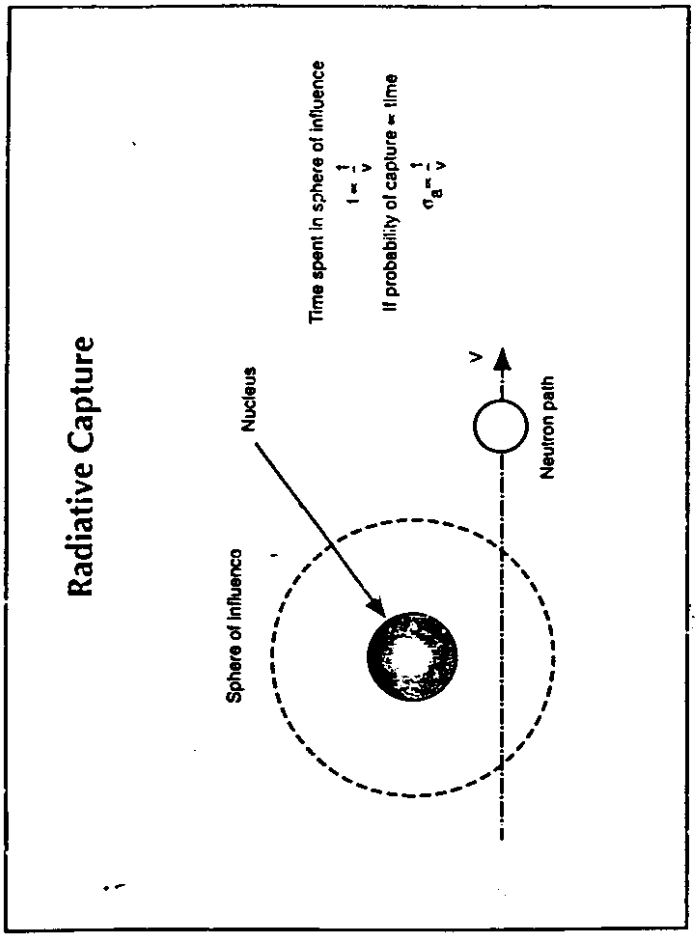
Time to react with nucleus = distance/velocity

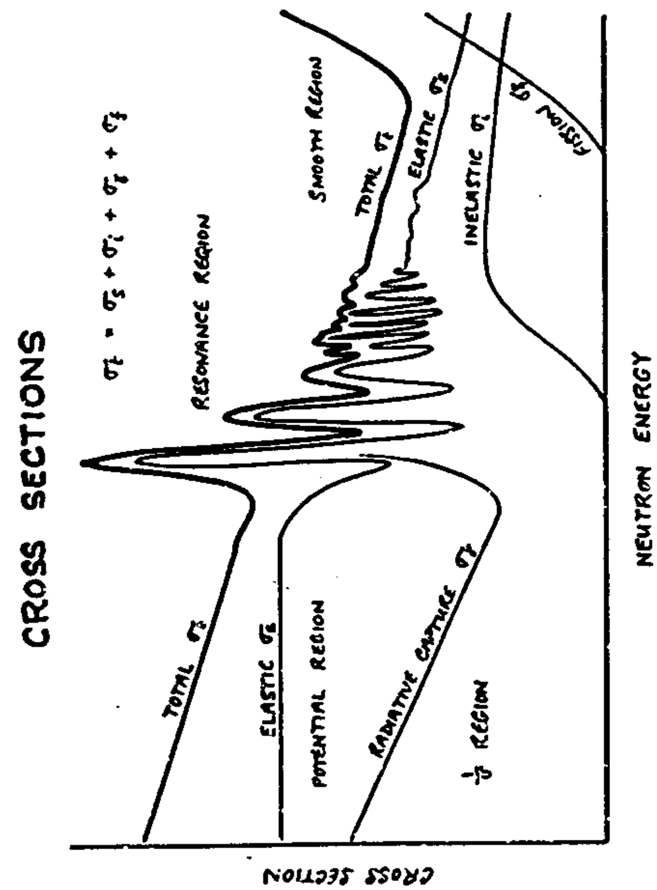
Time Model



Low velocity gives greater chance of capture

Energy Model





ENERGY LOSS IN SCATTERING

ENERGY AFTER COLLISION WITH NUCLEUS

$$E' = \frac{E}{(A+1)^2}\left[\cos\phi + \sqrt{A^2 - \sin^2\phi}\right]^2$$

MINIMUM ENERGY AFTER COLLISION WHEN $\phi = \pi$

$$E' = \left(\frac{A-1}{A+1}\right)^2 E = \alpha E$$

$$\alpha = \left(\frac{A-1}{A+1}\right)^2$$

AVERAGE ENERGY AFTER COLLISION

$$\bar{E}' = \tfrac{1}{2}(1+\alpha)E$$

AVERAGE ENERGY LOSS

$$\begin{aligned}\Delta E &= E - \bar{E}' \\ &= E - \tfrac{1}{2}(1+\alpha)E \\ &= \tfrac{1}{2}E - \tfrac{1}{2}\alpha E \\ &= \tfrac{1}{2}(1-\alpha)E\end{aligned}$$

AVERAGE FRACTIONAL ENERGY LOSS

$$\frac{\Delta E}{E} = \tfrac{1}{2}(1-\alpha)$$

MATERIALS OF LOW MASS NUMBER ARE MOST EFFECTIVE IN SLOWING DOWN NEUTRONS

Log Mean Energy Decrement

Logarithmic mean energy decrement

$$\begin{aligned}\xi &= \overline{\ln E_0 - \ln E} \\ &= \overline{\ln(E_0 / E)} \\ &= \overline{-\ln(E / E_0)}\end{aligned}$$

Value of ξ is given by

$$\xi = 1 + \frac{(A-1)^2}{2A}\ln\frac{(A-1)}{(A+1)}$$

Approximate value of ξ are given by

$$\xi = \frac{2}{A + 2/3}$$

ξ is Greek letter Xi

OH 4.3

Average Number of Elastic Collisions to Thermalize Fission Neutrons in Various Materials

H	18
D (Deuterium)	25
H_20 (Light water)	20
D_20 (Heavy water)	36
C-12 (Graphite)	115
U-238	2172

Slowing Down Powers and Moderating Ratios

	ξ	Σ_s(cm.$_{-1}$)(a)	$\xi\Sigma_s$	Σ_a	$\xi\Sigma_s/\Sigma_a$
He(b)	0.425	2×10^{-5}	9×10^{-6}	? very small	? large
Be	0.206	0.74	0.15	1.17×10^{-3}	130
H(c)	0.158	0.38	0.06	0.38×10^{-3}	160
BeO	0.174	0.69	0.12	0.68×10^{-3}	180
H_2O	0.927	1.47	1.36	22×10^{-3}	60
D_2O	0.510	0.35	0.18	0.33×10^{-4}(d)	5500(d)
D_2O	0.510	0.35	0.18	0.88×10^{-4}(e)	2047(e)
D_2O	0.510	0.35	0.18	2.53×10^{-4}(f)	712(f)

Definitions

Logarithmic mean energy decrement ξ

$N\xi = \ln \frac{E_i}{E_f}$

N = Number of Collisions

E_i = Initial energy (2 MeV)

E_f = Final Energy (0.025 eV)

Macroscopic scattering cross-section Σ_s

$\Sigma_s = N\sigma_s$

N = Nuclei per unit volume

σ = Microscopic cross-section

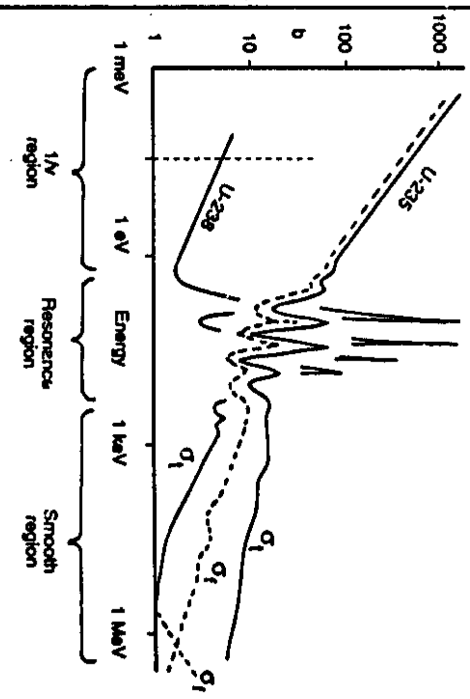
Slowing down power

$= \xi\Sigma_s$

Moderating ratio

$= \frac{\xi\Sigma_s}{\Sigma_a}$

Fission Characteristics



Interactions of importance

σ_s = Scattering
$\left.\begin{array}{l}\sigma_\gamma = \text{Radiative capture} \\ \sigma_f = \text{Fission}\end{array}\right\} \sigma_a = \text{Absorption}$

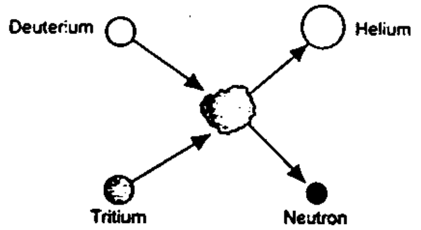
Capture/fission ratio: $\alpha = \sigma_\gamma / \sigma_f$

Probability of fission: $p = \sigma_f / \sigma_a$

Fusion and Fission

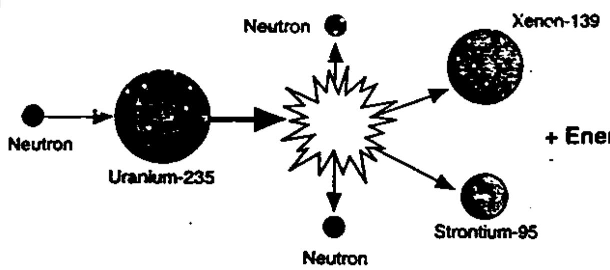
Fusion

$${}^2H_1 + {}^3H_1 \rightarrow {}^4He_2 + {}^1n_0$$

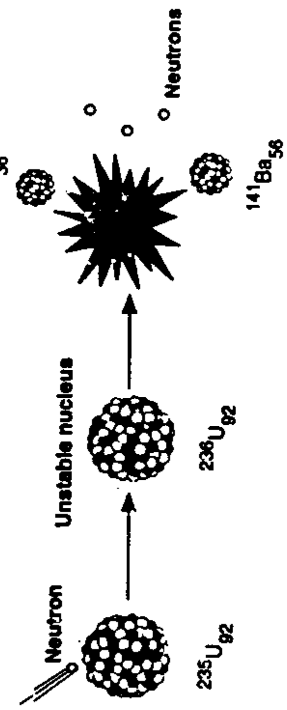


Fission

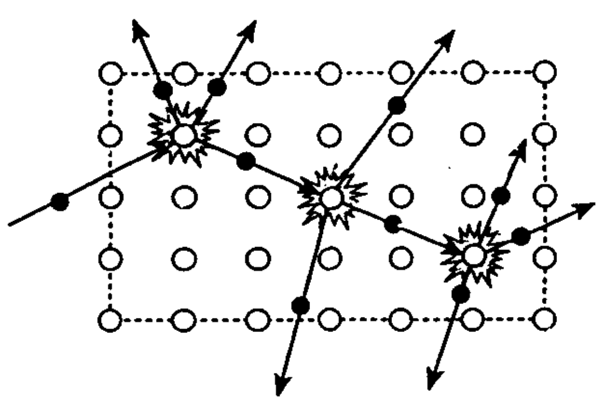
$${}^{235}U_{92} + {}^1n_0 \rightarrow {}^{139}Xe_{54} + {}^{95}St_{38} + {}^1n_0 + {}^1n_0$$



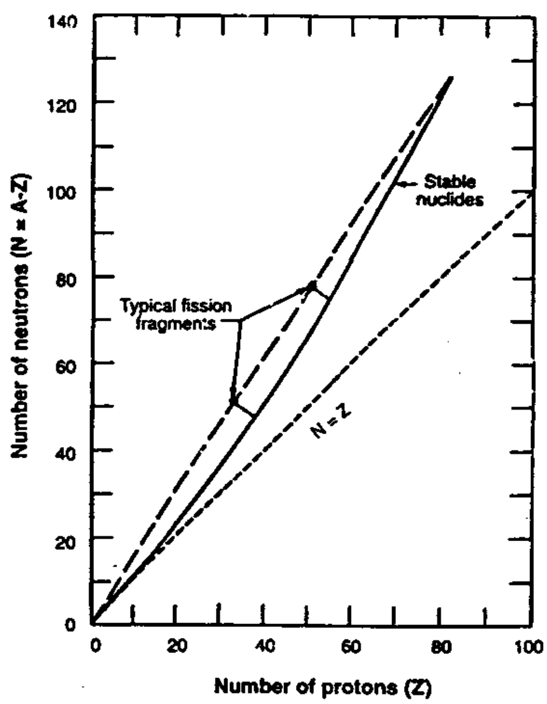
Fission Process



Fission Chain Reaction



Reason for Instability of Fission Fragments



Fission

Spontaneous Fission
- Rare but possible

Induced Fission
- Excited energy level must be above critical
- Adding a neutron adds energy to the nucleus (kinetic & binding energy)

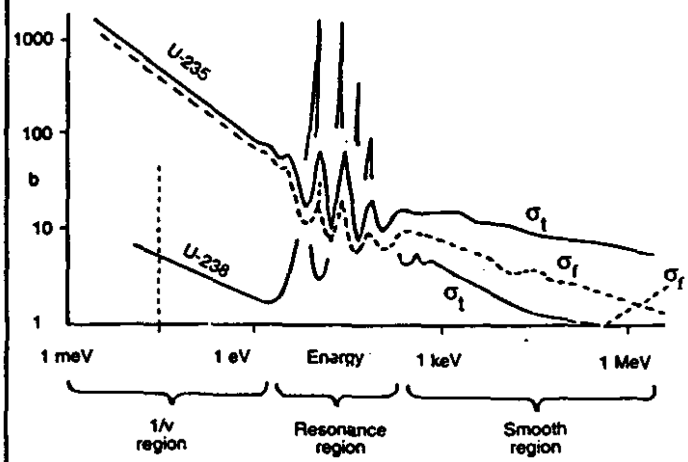
Fissile Nuclei
- Fission with zero energy neutrons

Fissionable Nuclei
- Fission with energetic (fast) neutrons only

Spontaneous Fission and Alpha Decay Rates of Uranium

	$t_{1/2}$ (α) (years)	$t_{1/2}$ (s.f.) (years)	α decay rate (atoms /s/kg)	s.f. decay rate (atoms /s/kg)
U-235	7.1×10^8	1.2×10^{17}	79×10^6	0.3
U-238	4.5×10^9	5.5×10^{15}	12×10^6	6.9

Fission Characteristics



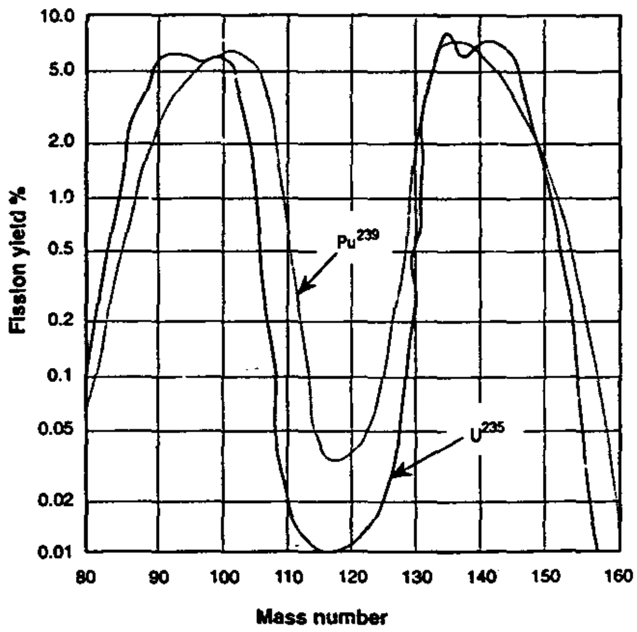
Interactions of importance

σ_s = Scattering
σ_γ = Radiative capture
σ_f = Fission
} σ_a = Absorption

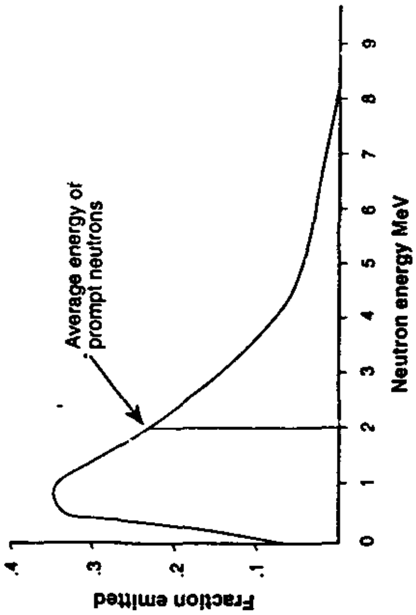
Capture/fission ratio: $\alpha = \sigma_\gamma / \sigma_f$

Probability of fission: $p = \sigma_f / \sigma_a$

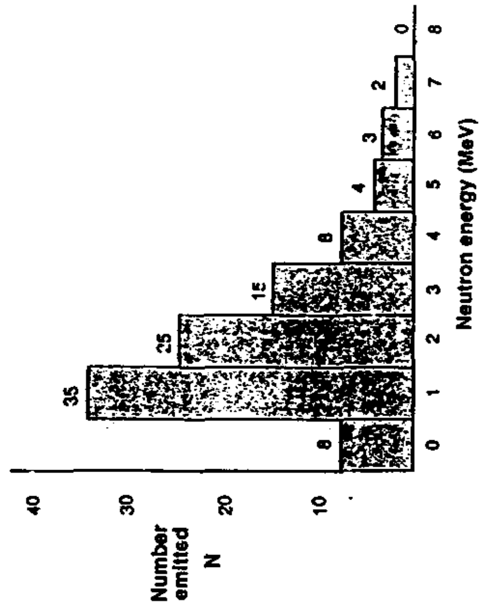
Fission Yield of U-235 and Pu-239



Energy Distribution of Prompt Fission Neutrons



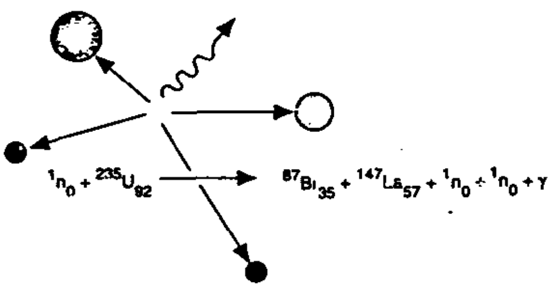
Energy Distribution
Sample of 100 Prompt Neutrons



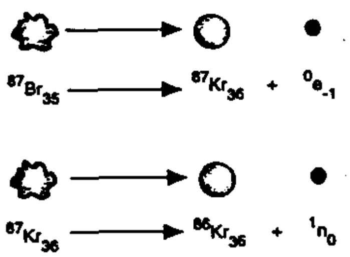
Delayed Neutrons

Delayed neutrons come from certain nuclei formed by beta decay of fission products

Fission:



Decay:



Fission Process Summary

* Critical energy of compound nucleus must be less than binding energy of added neutron

* Low energy neutrons interact more readily with U-235 to cause fission than do high energy neutrons. U-238, on the other hand, will only undergo fission with high-energy neutrons.

* Neutron/proton ratio curve results in additional neutrons being produced in fission ($\nu > 1$)

* Neutrons produced in fission have range of energies. $\bar{E} \approx 2$MeV.

* Neutrons must be slowed down to lower energies (thermalised or moderated) to start new cycle.